Name: ____________________ TA Name: ________________ Secret Word: ______________

Data 88S

April 19, 2024

1. Consider the following function f on the domain [0, 1]:

$$f(x) = \begin{cases} a & x \leq \frac{2}{3} \\ \frac{1}{x^2} & x > \frac{2}{3} \end{cases} \tag{1}$$

   Find the value of a that makes this function a valid probability density function over [0, 1].

2. Suppose X_1, X_2, X_3 are i.i.d. uniform over [0, 1]. Let Y = $\max(X_1, X_2, X_3)$.
   (a) Find the probability density function of Y

   (b) Find $P(Y < \frac{1}{3})$

   (c) Find $P(Y \geq \frac{1}{2})$

   (d) Find $E[Y]$

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Chapter 10, Exercise 1

3. Let $X_1, X_2, X_3, ...$ be i.i.d. with density given by

$$f(x) = \begin{cases} 0 & x \leq 50 \\ \frac{c}{x^4} & x > 50 \end{cases} \tag{2}$$

This is one of the Pareto densities, sometimes used in economics to represent distributions of wealth in populations where a small percent of the population owns a large percent of the wealth.

(a) Find c.

(b) Find the cdf of X_1 and sketch its graph.

(c) Find $E(X_1)$.

(d) Find $Var(X_1)$.

Chapter 10, Exercise 2

4. A class starts at 3:10 p.m. Seven students in the class arrive at random times $T_1, T_2, ..., T_7$ that are i.i.d. with the uniform distribution on the interval 3:07 to 3:12.

(a) Find $E(T_1)$

(b) What is the chance that all seven students arrive before 3:10?

(c) Let $X = max(T_1, T_2, ..., T_7)$ be the time when the last of the seven students arrives. Find the cdf of X.