Name: ____________________ TA Name: ________________ Secret Word: ______________

Data 88S

April 5, 2024

Law of Averages

1. A fair coin is tossed repeatedly. For each win condition, which number of tosses gives you a higher chance of winning: 10 tosses or 100 tosses?

   In each of parts below, pick one of the two options without calculation and explain your choice.

   (a) You win a prize if there are at least 60% heads.

   (b) You win a prize if there are at least 40% heads.

   (c) You win a prize if there are between 40% and 60% heads, inclusive.

   (d) You win a prize if there are exactly 50% heads.

Weak Law of Large Numbers

2. (a) City A has a population of 4 million, and City B has a population of 400,000. In City A, 60,000 people hold degrees in statistics (1.5% of the population), and in City B, 80,000 people hold degrees in statistics (20% of the population). Anton, a surveyor in City A, selects a simple random sample of 2,000 people from the city. Borong, a surveyor in City B, selects a random sample with replacement, also of 2,000 people from their city, independent of Anton's sample. Let X_A be the number of people in Anton's sample who hold degrees in statistics, and X_B be the number of people in Borong's sample with degrees in statistics. Find $E(X_A + 10X_B)$ and $Var(X_A + 10X_B)$.

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(b) The Bureau of Statistics in City A has just now received millions of dollars in an unexpected donation! Anton now has the budget to draw another $n-1$ simple random samples from the population of City A and all of the samples are independent of one another. Let A_n denote the average number of people in Anton's n samples with degrees in statistics. What does the Weak Law of Large numbers imply? Select all that apply.

i. $P\left(|A_n - E(A_n)| < c\right) \to 1$ as $n \to \infty$, for any fixed $c > 0$.

ii. $P\left(|A_n - E(A_n)| < c\right) \to 0$ as $n \to \infty$, for any fixed $c > 0$.

iii. $P\left(|A_n - E(A_n)| > c\right) \to 0$ as $n \to \infty$, for any fixed $c > 0$.

iv. $P\left(A_n \in [E(A_n) - c, E(A_N) + c]\right) \to 1$ as $n \to \infty$, for any fixed $c > 0$.

CLT Exploration

3. go to https://tinyurl.com/88sdemo and use the notebook to simulate the distribution of a sample sum.