

# Data 88S

March 13, 2024

## Chapter 5, Exercise 12

1. A coin is tossed four times. Let  $X$  be the number of heads in the first three tosses and  $Y$  the number of heads in the last three tosses.
  - (a) The distribution of  $Y$  is one of the famous ones. Provide its name, parameters, and expectation.
  - (b) For each possible value  $x$  of  $X$ , find  $E(Y|X = x)$ . Your answer to 5c will be helpful.
  - (c) Find  $E(Y)$  by conditioning on  $X$ , and confirm that the result is the same as your answer to Part a.

## Chapter 5, Exercise 13

2. A survey organization in a town is studying families with children. Among the families that have children, the distribution of the number of children is as follows.

|  |     |     |     |      |      |
|--|-----|-----|-----|------|------|
| <b>Number of Children <math>n</math></b>       | 1   | 2   | 3   | 4    | 5    |
| <b>Proportion with <math>n</math> Children</b> | 0.2 | 0.4 | 0.2 | 0.15 | 0.05 |

Suppose each child has chance 0.51 of being male, independently of all other children. What is the expected number of male children in a family picked at random from those that have children?

**Chapter 5, Exercise 10**

3. Five thousand votes have been cast in an election. Each vote is either for Candidate A or for Candidate B. The voting is over but the results have not yet been announced.

A polling organization wants to estimate the margin of victory for Candidate A. This parameter is defined as the proportion of votes for Candidate A minus the proportion of votes for Candidate B. Note that this margin of “victory” could be negative if Candidate B gets more votes than Candidate A.

The polling organization conducts an exit poll by surveying a simple random sample of 100 voters after all of the 5000 voters have finished voting.

- (a) Let  $p$  be the proportion of the 5000 voters who voted for Candidate A. Express the margin of victory in terms of  $p$ .
- (b) Let  $X$  be the number of sampled voters who voted for Candidate A. Use  $X$  to construct an unbiased estimator of the margin of victory.
4. Bella chooses an integer  $X$  uniformly at random from 1 to 425. She then chooses an integer  $Y$  uniformly at random from  $1, \dots, X$ . Find  $E(Y)$ .
5. Bella chooses an integer  $N$  at random from  $Pois(\mu)$ . She then picks  $N$  cards from a deck with replacement. Find the expected number of ace cards.