Data 88S

March 1, 2024

Chapter 5, Exercise 8

1. A sports team consists of n players. Each player has a backpack that from the outside looks the same as all the others'. Suppose each player picks up a backpack at random without replacement.

   (a) Fix an integer k in the range 1 through n. What is the chance that Player k picks up her own backpack?

   (b) Let B be the number of players who pick up their own backpacks. Find $E(B)$.

   (c) For large n, approximately what is the distribution of B? Why?

   (d) Are your answers to parts b) and c) consistent?

2. Suppose $X \sim Bernoulli(0.6), Y \sim Bernoulli(0.8)$

   (a) Write down a joint distribution assuming that X and Y are independent.

   (b) What is the distribution of $X + Y$?

   (c) What is the expectation of $X + Y$? (calculate it from the above distribution)

$\mathbb{P}$	$Y = 0$	$Y = 1$
$X = 0$	0.1	0.3
$X = 1$	0.1	0.5

(d) From now on, assume that X and Y have the following distribution:
Are X and Y independent? What are the marginals of X and Y?

(e) Now what is the distribution of $X + Y$?

(f) Now what is the expectation of $X + Y$? Does the result surprise you?

Chapter 5, Exercise 7

3. A randomized controlled experiment has 100 participants of whom 40 are women. A simple random sample of 70 participants is assigned to the treatment group and the remaining 30 are in the control group.

How many more women are expected to be in the treatment group than in the control group?

Chapter 5, Exercise 4

4. In roulette, the bet on a "split" pays 17 to 1 and there are 2 chances in 38 to win. "Pays 17 to 1" means that if a gambler bets one dollar on a split and wins the bet, then her net gain will be 17 dollars. If she loses the bet, then she loses her dollar; that is, her net gain is -1 dollars.

(a) Suppose 200 gamblers bet on a split. Find the expectation of their total net gain.

(b) Does your answer to Part a depend on whether the 200 bets were on the same or different spins of the roulette wheel?