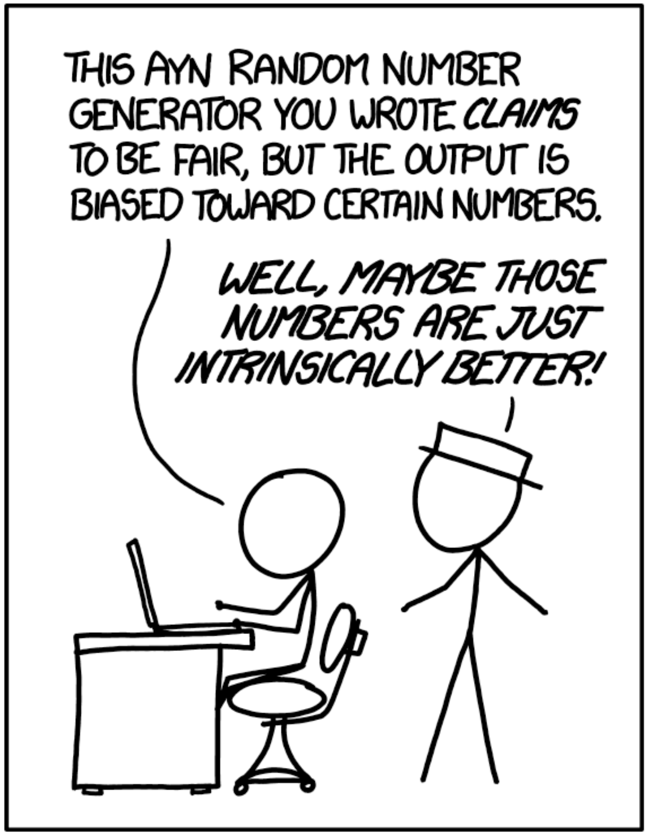
Stat 88: Probability & Math. Stat in Data Science



https://xkcd.com/1277

Lecture 8: 2/2/2024

Random variables & their distributions, and a special distribution

3.1, 3.2, 3.3

Shobhana Stoyanov

Agenda

- Counting permutations and combinations
- Random variables and their distributions
- The binomial distribution

Counting permutations

each action i has n_i

- Recall the product rule of counting, where we counted number of outcomes when we had a sequence of k actions, each with n_i outcomes, so the total number of outcomes is $n_1 \times n_2 \times \ldots n_k$
- # of ways to rearrange n things, taking them 1 at a time is $n!$
- If we have only $k \leq n$ spots to fill, then $n \cdot (n-1) \cdot \ldots \cdot \left(n-(k-1)\right)$
- # of perm. of n things taken k at a time.

$n!/(n-k)!$

$\underbrace{\underset{1st}{22} \cdot \underset{2nd}{21} \; \underset{3rd}{20} \; 19 \; \ldots \; \underset{k^{th}}{\_\_}}$

- Count the number of sequences of 3 letters taken from the English alphabet without replacement. _ · _ · _

11 field positions
soccer

22 in roster

$22 \cdot 21 \cdot 20 \cdot \ldots 13 \cdot 12$

$= \dfrac{22!}{11!}$

$= 22!$

Counting combinations

$(22-11)!$

$\frac{26!}{23!}$ $\_\ \_\ \_$ we count $\{a\ b\ c\}$ # $\{b\ c\ a\}$

- Suppose we don't care about the sequence but just *which* letters were chosen (so *abc* = *bca* = *cab* etc.) Then all of these combinations count as 1 selection. We need to take the number we got above and divide by the number of arrangements of 3 letters $= \underline{26}\cdot\underline{25}\cdot\underline{24} = \frac{26!}{23!} = \frac{26!}{(26-3)!}$
- If we don't care about order, then we are counting subsets, and this number is denoted by $\binom{n}{k}$ (read as "n choose k") which we get by dividing: $n \cdot (n-1) \cdot \ldots \cdot (n-(k-1))$ by $k!$, so $\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

${}^nC_k$ $\quad C^n_k$

a, b, c

$\underline{3}\cdot\underline{2}\cdot\underline{1} = 6 = 3!$

- Note: $\binom{n}{n} = 1$, $\binom{n}{0} = 1$
- $\left(\frac{n!}{(n-k)!}\right) \div k! = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$

Examples

of card hands of 5 cards $= \binom{52}{5}$

Let's consider poker, in which each player is dealt 5 cards. How many hands of 5 cards are possible from a standard deck? Recall that a standard deck has 52 cards, consisting of 4 suits (♥, ♦, ♣, ♠) of 13 cards each (**2, 3, …,10, J, Q, K, A**)

Ω = all 5-card hands

- Chance of a pair in poker= $\dfrac{\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$

- Chance of two pairs = $\dfrac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\cdot\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$

$\frac{52}{52}\cdot\frac{3}{51}\left(1-\frac{2}{50}\right)^3$

Chocolate.

- Chance of "full house in poker" =

3 cards of 1 kind & 2 of another

13 kinds

$$\binom{13}{2} = \frac{13!}{(13-2)!\,2!} = \frac{13\cdot 12\cdot \cancel{11!}}{\cancel{11!}\,2!}$$

$$= \frac{13\cdot 12}{2!}$$

$$\binom{13}{1} = 13$$

$$\binom{n}{0} = 1 = \binom{n}{n}$$

$$\binom{13}{0} = 1$$

$\boxed{0! = 1}$ Fact

$$\binom{13}{1} = 13 \qquad \binom{12}{1} = 12$$

$$\binom{13}{2} \overset{?}{=} \binom{13}{1}\binom{12}{1}$$

$$\frac{13\cdot 12}{2\cdot 1} = 13\cdot 12$$

4 suits

Section 3.1: Vocabulary

- When we have two kinds of tickets in a box and we draw tickets at random from this box, each draw is called a ***trial***
- We call the two kinds (binary) of outcomes ***Success***, and ***Failure***
- Might be with replacement (like a coin toss) or without replacement (drawing a sample of voters from a city and checking number of registered voters)
- Read about Paul the octopus and Mani the parakeet and their soccer predictions
- Note that Paul made 8 correct 2010 WC predictions. What is the chance of 8 correct if picking completely at random? (like tossing a coin and getting all heads)

Please read
about Paul.

Back to counting outcomes of tosses

- Toss a coin 8 times, how many possible outcomes?
- What is the chance of ***all*** heads?
- If each of the students in this class present today flip a coin 8 times, what is the chance that ***at least 1 person*** gets all heads?

2^{nd} exercise

Ω = outcomes of tossing a fair coin 3 times

h	Prob(h)
0	1/8
1	3/8
2	3/8

Let h = # of Heads in 3 tosses

3	$1/8$

Challenge 1 Do it for # of Heads in 4 tosses

Challenge 2 If Prob of heads on any toss is $1/3$. then how does your table change?

3.2 Random Variables

- A real number – we don't know exactly *what* value it will take, but we know the possible values.

- The number of heads when a coin is tossed 3 times could be 0, 1, 2, or 3.
- The sum of spots when a pair of dice is rolled could be 2, 3, 4, 5, …, 12.

- These are both examples of *random variables*.
- *Variable* because the number takes different values
- *Random variable* because the outcomes are not certain.