P-value is a conditional distribution "$P(\text{data}|H_0)$"

Stat 88: Prob. & Mathematical Statistics in Data Science

https://xkcd.com/892/

Lecture 33 : 4/15/2024 (PART 1)

Section 9.3, 9.4

A/B testing ~~& confidence intervals~~

A/B testing: comparing *two* distributions

- Data 8, section 12.3, randomized controlled trial to see if botulinum toxin could help manage chronic pain.
- 31 patients → 15 in treatment group, 16 in control group. 2 patients in the control group reported pain relief and 9 in the treatment group.

- A/B testing is a term used to describe hypothesis tests which involve comparing the distributions of *two* random samples. (Earlier we had *one* sample and made a hypothesis about its distribution.)
- In particular, we can conduct an A/B test for hypothesis tests involving results of randomized controlled trials, A is the control group and B the treatment group.

Trt A 15 Ctrl B 16

31 tickets

T/1 tkt 1, T/0 tkt 2, C/0 . . . — — — C/1 tkt 31 ← orig.

C/1, T/0, C/0 — — — T/1

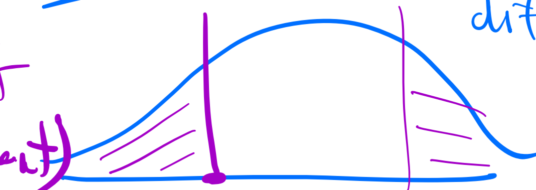
Fisher's exact test

- Control group: 16 patients, 2 reported relief
- Treatment group: 15 patients, 9 reported relief

Total # of patients reporting pain relief → 11 (out of 31)

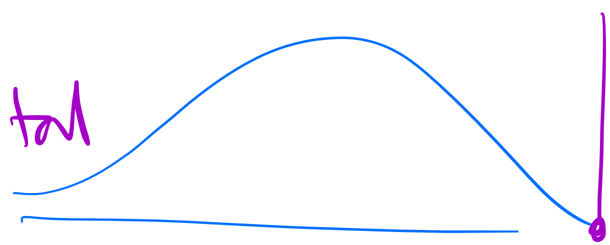
- H_0: The treatment has no effect (there would have been 11 patients reporting pain relief no matter what, and it just so happens that 9 of them were in the treatment group)
- H_A: The treatment has an effect

Perm test: shuffle labels & recompute diff in prop.



Test statistic be the # of patients that reported pain relief in sample (Treatment)

If H_0 is true then X is just the # of successes in a SRS of size 15 out of 31 total with 11 successes in pop. of 31.



$X \sim HG(N=31, G=11, n=15)$ using H_0.

$\mathbb{E}(X \mid H_0) = 15 \cdot \frac{11}{31} \approx 5.32$

Now let's use test statistic $T = X - 5.32$, observed value of $T = 9 - 5.32 = 3.68$

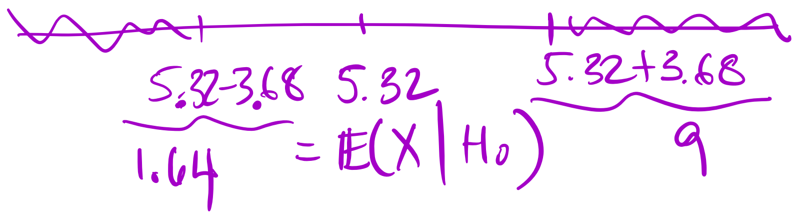
p-value $= P(|T| \geq 3.68) = P(X \leq 5.32 - 3.68)$
$+ P(X \geq 5.32 + 3.68)$

obs. diff is 3.68

$= P(X \leq 1.64) + P(X \geq 9)$

$= P(X \leq 1) + P(X \geq 9)$

≈ 0.00915



Reject H_0; the trt (appears) to have an effect.

Exercise for multiple chocolates

Example: The Lady Tasting Tea

- The first person to describe this sort of hypothesis test was the famous British statistician Ronald Fisher. In his book *The Design of Experiments,* he describes a tea party in which a lady of his acquaintance claimed that she could tell from tasting a cup of tea if the milk had been poured first or the tea.
- Fisher immediately set up an experiment in which she was given multiple cups of tea and asked to identify which of them had had the tea poured first. She tasted 8 cups of tea, of which 4 had the tea poured first, and identified 3 of them correctly. Does this data support her claim?

Actually happened

Lady says

	Tea first	Milk first
Tea first		
Milk first		

Exercise

Example: Gender bias ?

- Rosen and Jerdee conducted several experiments using male bank supervisors (this was in 1974) who were given a personnel file and asked to decide whether to promote or hold the file. 24 were randomly assigned to a file labeled as that of a male employee and 24 to a female.
- 21 of the 24 males were promoted, and 14 of the females. Is there evidence of gender bias?

Moving on to Chapter 10

Chapter 10: Probability Density

Earlier we saw discrete random variables that took particular discrete values on $\mathbb{R}$ and had discrete prob. mass functions.

Now instead of restricting the r.v. to particular values, we allow it to take any value in a given interval.

For example we might have a r.v. X s.t. X is in $[0,1]$ $\left(X \in [0,1] \rightarrow 0 \leq X \leq 1 \right)$

Instead of a prob. mass function, we will define a PROBABILITY DENSITY function.

Imagine the total prob. mass of 1 is smeared over the interval according to the density function. Such a r.v. whose probabilities are defined this way is called a

CONTINUOUS RANDOM VARIABLE

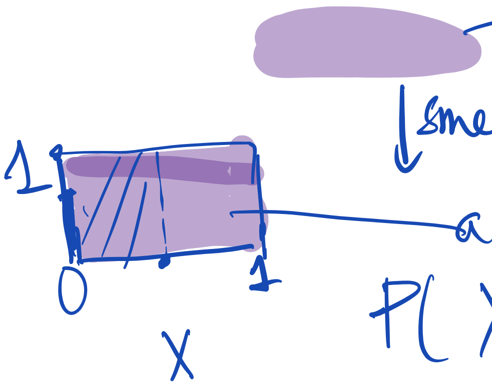
& the probabilities are computed by areas.

mass of 1

smeared across [0,1]

Total area is 1

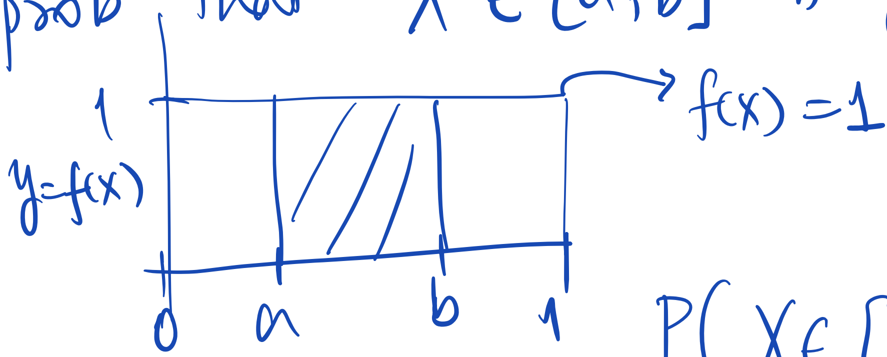
area



$$P(X \in [0,1]) = 1$$

$$P(X \in [0, \tfrac{1}{2}]) = \frac{1}{2}$$

If the density function is flat, we call X a uniform r.v. & the prob that $X \in [a,b]$ is just the area of rectangle whose base is interval $[a,b]$



$$P(X \in [a,b]) = b - a$$

$$P(X \in [0, \tfrac{1}{2}]) = \frac{1}{2} \qquad P(X \in [\tfrac{1}{4}, \tfrac{3}{4}]) = \frac{1}{2}$$

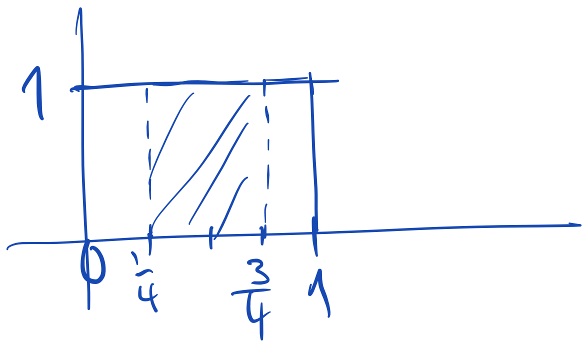
$$P(X \in [\tfrac{1}{2}, 1]) = \frac{1}{2}$$

If X is a uniform r.v. (density function is flat)

then the only thing that matters is width of the interval, not where it is in $[0,1]$

$$P\left(\tfrac{1}{4} < X < \tfrac{3}{4}\right) = P\left(X \in \left(\tfrac{1}{4}, \tfrac{3}{4}\right)\right)$$

$$= \tfrac{1}{2} = P\left(X \in \left[\tfrac{1}{4}, \tfrac{3}{4}\right]\right)$$

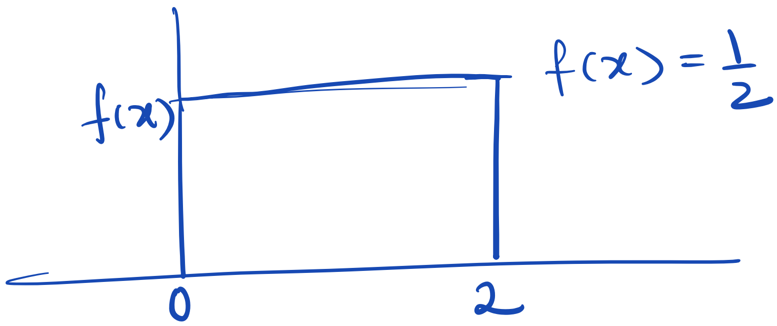


Define a prob. density function $f(x)$

by ① $f(x) \geq 0$

② $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$

Cdf is defined to be $F(x) = \displaystyle\int_{-\infty}^{x} f(t)\,dt$



$$f(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq 2 \\ 0 & o/w \end{cases}$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{x} \frac{1}{2}\,dt$$

$$F(1.2) = P(X \leq 1.2) = \int_{-\infty}^{1.2} \frac{1}{2}\,dt = \int_{0}^{1.2} \frac{1}{2}\,dt = \frac{1.2}{2} = 0.6$$