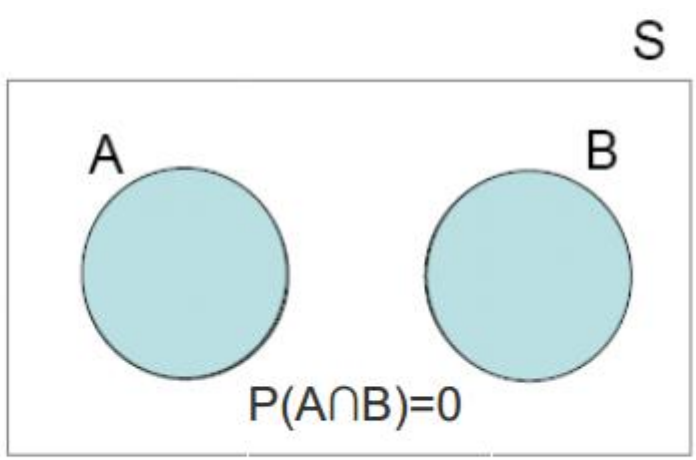
Probability and Mathematical Statistics in Data Science

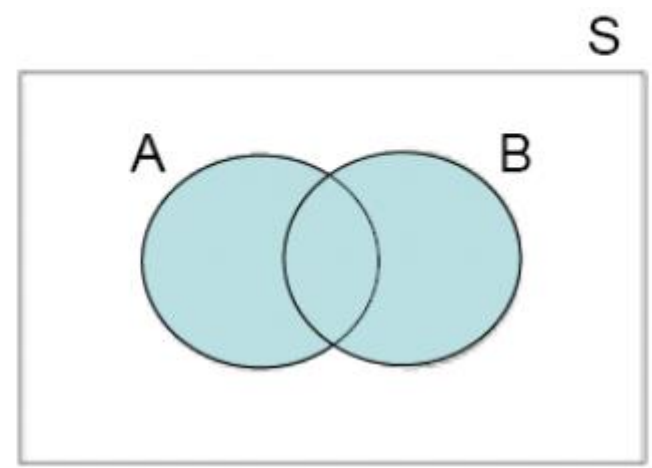
Lecture 03: Section 2.1: Chance of an Intersection

More Probability Properties

Consider an experiment whose sample space is S. For each event A (B) in S, we assume that a number P(A) is defined and satisfies the following rules:

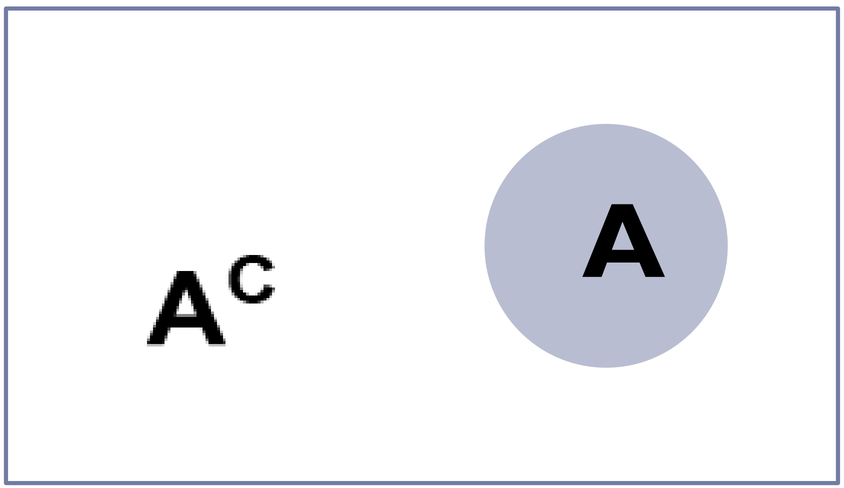
- 1. $0 \leq P(A) \leq 1$.
- 2. $P(A^c) = 1 - P(A)$.
- 3. If A and B are disjoint (or mutually exclusive), then $P(A \cup B) = P(A) + P(B)$.
- 4. For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

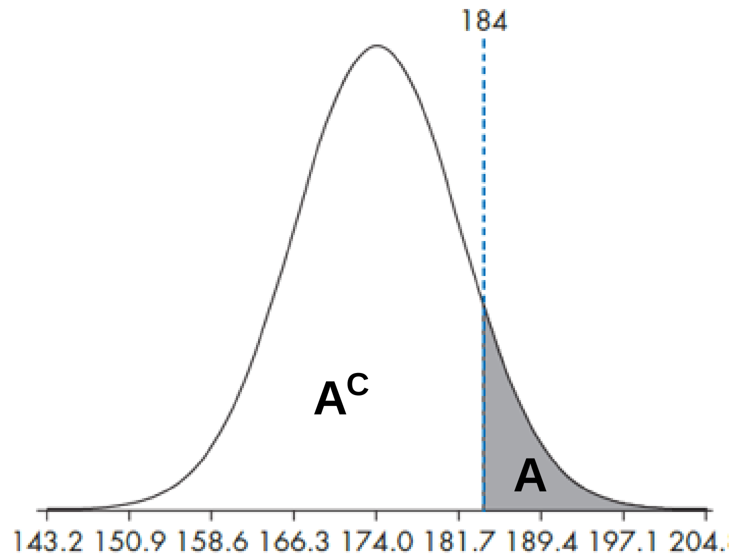




Complement Rule

- The set of outcomes that are *not* in the event **A** are called the complement of **A**, denoted $\mathbf{A}^{C}$**.**
- The probability of an event occurring is 1 minus the probability that it doesn't occur: $P(\mathbf{A}) = 1 - P(\mathbf{A}^{C})$

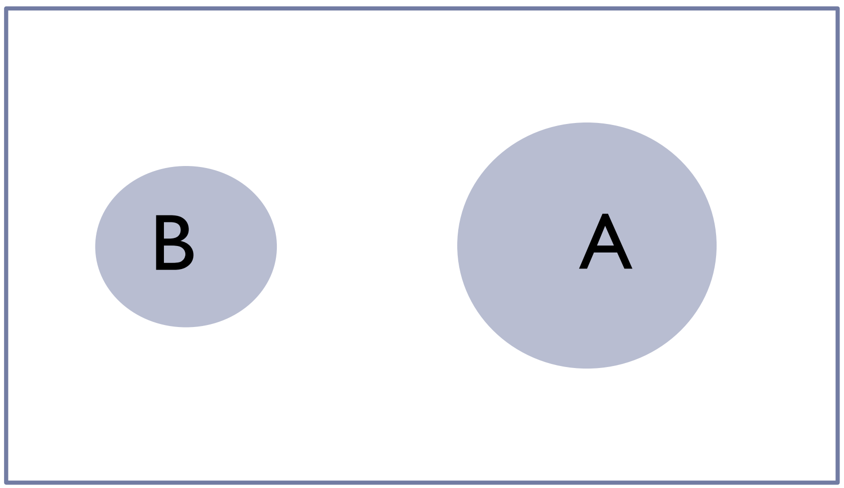


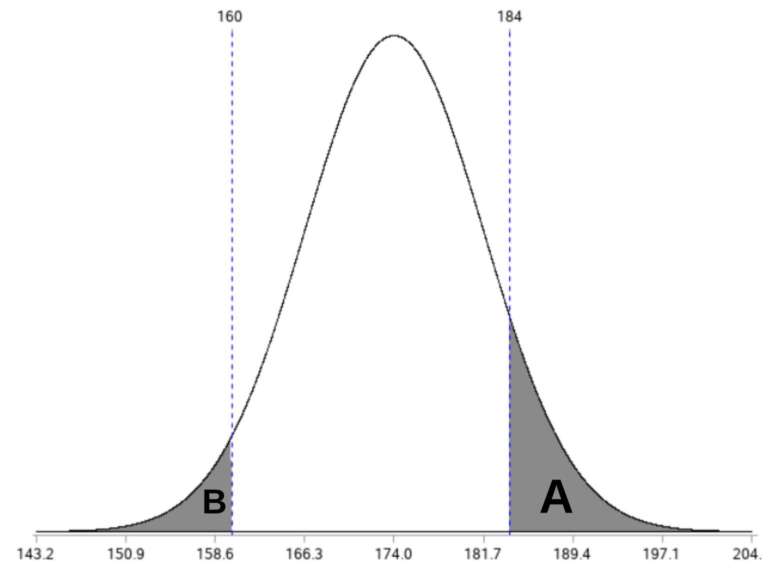


- **Example:** Let A be the probability an man is taller than 184 cm.. P(A) = 0.10 then what is the $P(\mathbf{A}^{C})$?

Addition Rule

- Events that have no outcomes in common are called disjoint events (or mutually exclusive events).





- *P*(A or B) = *P*(A) + *P*(B), when A and B are disjoint events.

- Note: *P*(A or B) can be written as *P*(A ∪ B)

The General Addition Rule

- When two events **A** and **B** are disjoint, we can use the addition rule for disjoint events:

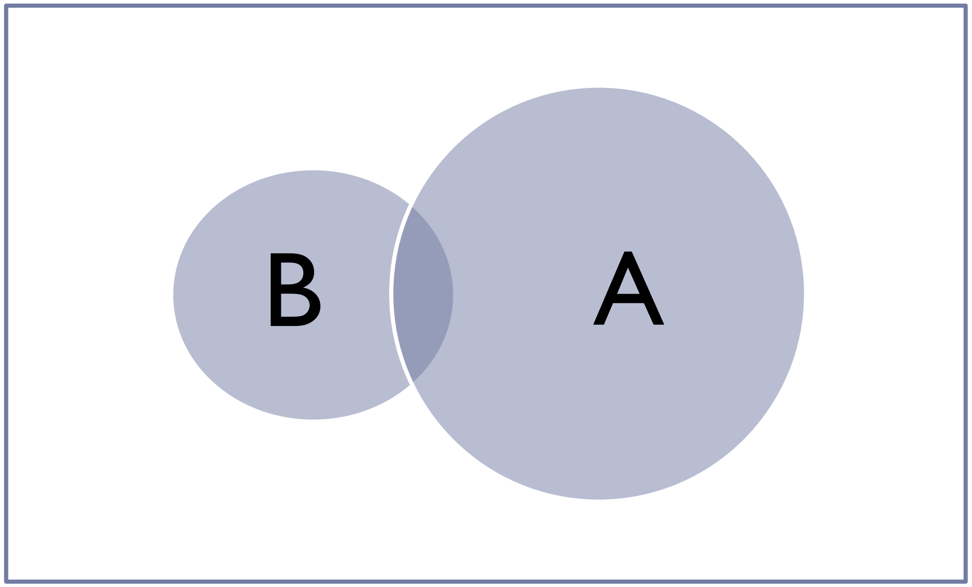
$$P(\text{A or B}) = P(\text{A}) + P(\text{B})$$

$$\textit{or } P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B})$$

- However, when our events overlap, the addition rule will count the probability of *both* **A** and **B** occurring twice.

- Therefore, we need the **General Addition Rule**.

The General Addition Rule



For any two events **A** and **B**,

P(A or B) = P(A) + P(B) – P(A and B)

or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

The General Addition Rule

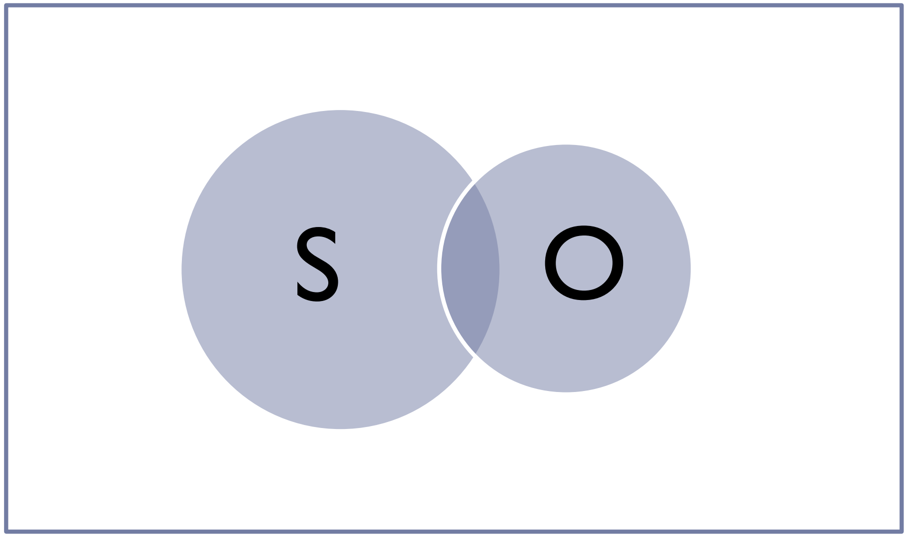
Example

- Hospital records show that 14% of all patients are admitted for surgical treatment, 8% are admitted for obstetrics, and 2% receive both obstetrics and surgical treatment.

- If a new patient is admitted to the hospital, what is the probability that the patient will be admitted either for surgery, obstetrics, or both?

- When thinking about answers to probability question, it is often helpful to think in terms of the finite.

The General Addition Rule: Example



- S="surgery"; O="obstetrics";
- P(S)=0.14; P(O)=0.08; P(S and O)=0.02
- P(S or O) = P(S) + P(O) – P(S and O)= 0.14 + 0.08 - 0.02 = 0.20

- Let's say the hospital had 1000 patients. We can say that 200 of those patients were there for surgery or obstetrics (or both).

Example

A store accepts either VISA or Mastercard. 50% of the stores customers have VISA, 30% have Mastercard and 10% have both.

What is the probability that a customer has a credit card the store accepts?

Let A = customers has VISA

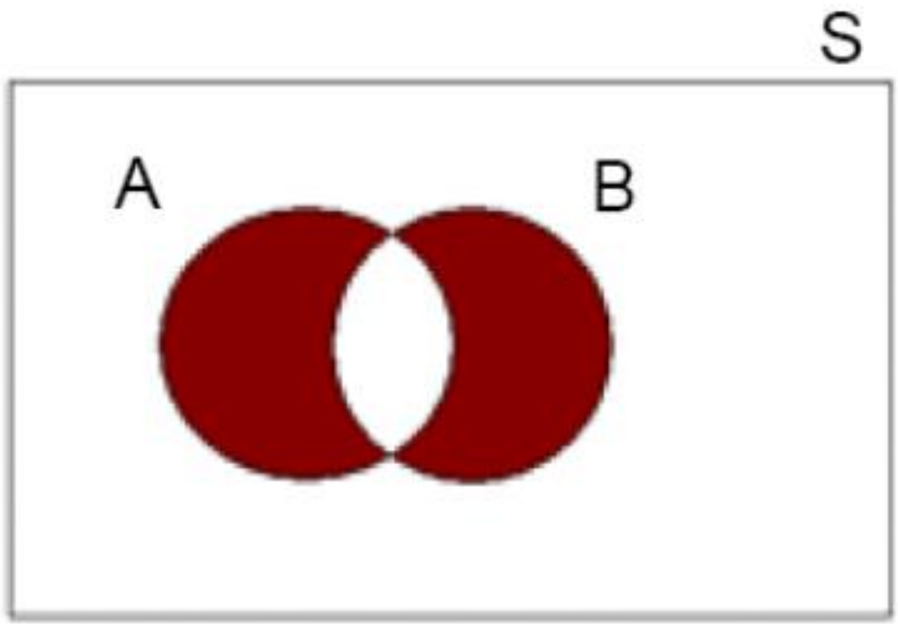
Let B = customers has Mastercard

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.3 - 0.1 = 0.7$$

Example cont.

What is the probability that a customer has either a VISA or MC, but not both?



P(A or B but not both) = P(A) + P(B) – 2P(A∩B)

= 0.5 + 0.3 – 0.2 = 0.6

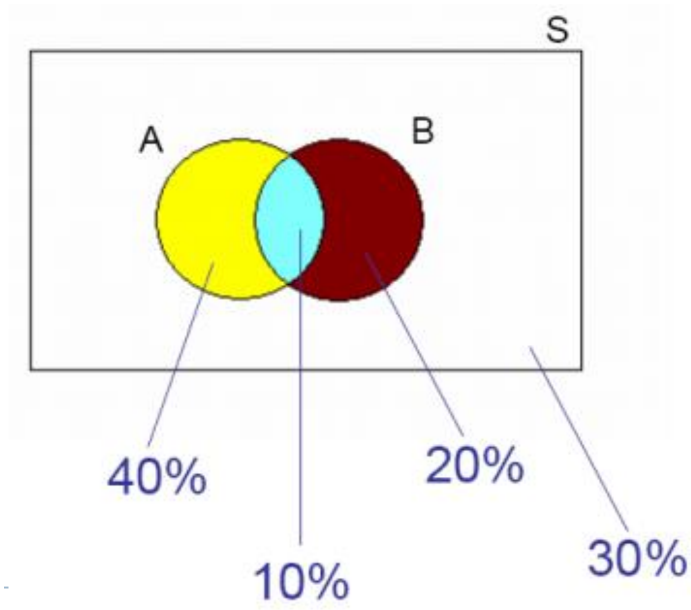
Example cont.

What is the probability that a customer has a VISA but no MC?

- P(A but not both) = P(A) – P(A∩B) = 0.5 – 0.1 = 0.4

What is the probability that a customer has a MC but no VISA?

P(B but not both) = P(B) – P(A∩B) = 0.3 – 0.1 = 0.2



The Multiplication Rule

- Conditional probability written as P(B|A), read as "the probability of the event B, given that the event A has occurred"

- Chance that two things will both happen is the chance that the first happens, ***multiplied*** by the chance that the second will happen *given* that the first has happened.

- For two events **A** and **B** that are **not independent**,

$$P(\text{A } and \text{ B}) = P(\text{A}) \times P(\text{B}|\text{A})$$

Multiplication rule

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

- Ex.: Draw a card at random, from a standard deck of 52
  - P(King of hearts) =?
- Draw 2 cards one by one, **without replacement**.
  - P(1st card is K of hearts)=
  - P(2nd card is Q of hearts| 1st is K of hearts) =
  - P(1st card is K of hearts AND 2nd is Q of hearts) =

Independence and The Multiplication Rule

- Two events, A and B, are considered **independent** if the fact that A occurs does not affect the probability of B occurring

- When two events **A** and **B** are independent:

$$P(\text{A and B}) = P(\text{A}) \times P(\text{B})$$

Note: P(A and B) can be written as $P(\text{A} \cap \text{B})$

The verification of this rule is as follows:

$$P(\text{A } \textit{and} \text{ B}) = P(\text{A}) \times P(\text{B}|\text{A}) = P(\text{A}) \times P(\text{B})$$

With Replacement or Without Replacement

Q. You toss a fair coin two times. What is the probability of obtaining two heads?

Q. You pick three cards at random from a deck **with replacement.** Find the probability of each event described below.

a) You get no aces.

b) You get all hearts.

c) The third card is your first red card.

With replacement ensures independence

Example: Without Replacement

- Deal 5 cards from the top of a well shuffled deck. What is the probability that all are hearts? (Extend the multiplication rule)

- Deal 5 cards, what is the chance that they are all the same suit?