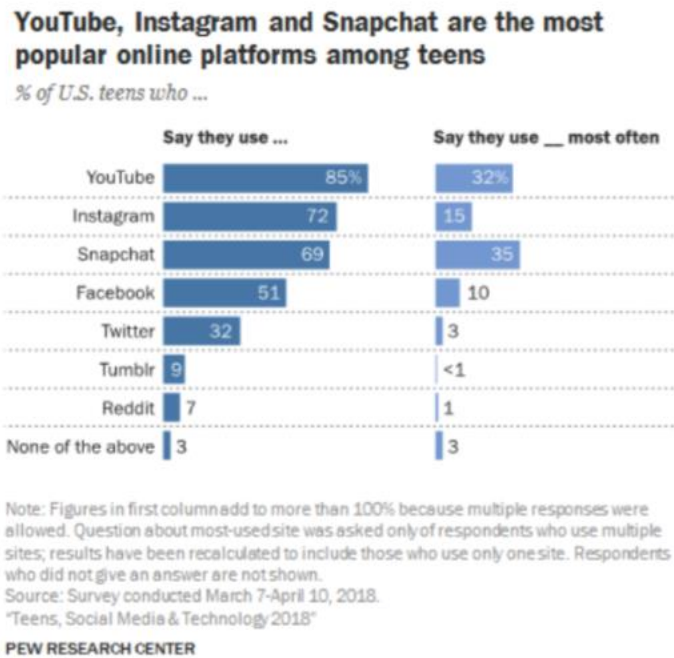
Probability and Mathematical Statistics in Data Science

Lecture 02 – Section 1.2: Exact Calculations and Bounds
Section 1.3: Fundamental Rules

Exact Calculations, or Bound?



3. What is the chance that FB *or* Twitter was their favorite?

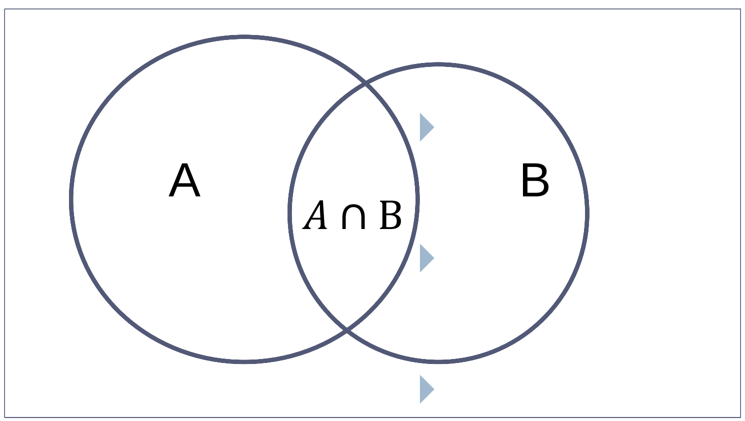
What was the answer?

What can you say about the chance that a randomly selected teen used FB or Twitter (not necessarily most often)?

Example with bounds

- When we get some information about the outcome or event whose probability we want to figure out, our outcome space reduces, incorporating that information.

- Bounds on probabilities of unions and intersections when events are *not* mutually exclusive.



P(A) = 0.7, P(B) = 0.5

$$\_\_\_ \leq P(A \cup B) \leq \_\_\_$$

$$\_\_\_ \leq P(A \cap B) \leq \_\_\_$$

Defining Events

Example: Flip a coin twice. Four possible outcomes, S={HH, HT, TH, TT}.

- Let A be the event that we obtain at least one H in the two flips. A={HH, HT, TH}.
- What is the P(A)?
- Let B be the event that we obtain two H's in the two flips. B={HH}.
- What is the P(B)?

Events and Sets

In a more abstract way, we can think about Sample Space, Outcomes and Events in terms of the Set Theory.

- Outcomes are the objects (elements)
- Events are sets (collections of elements)
- Sample Space is the whole set (collection of all elements)

We will treat events and sets synonymously.

Set Operations

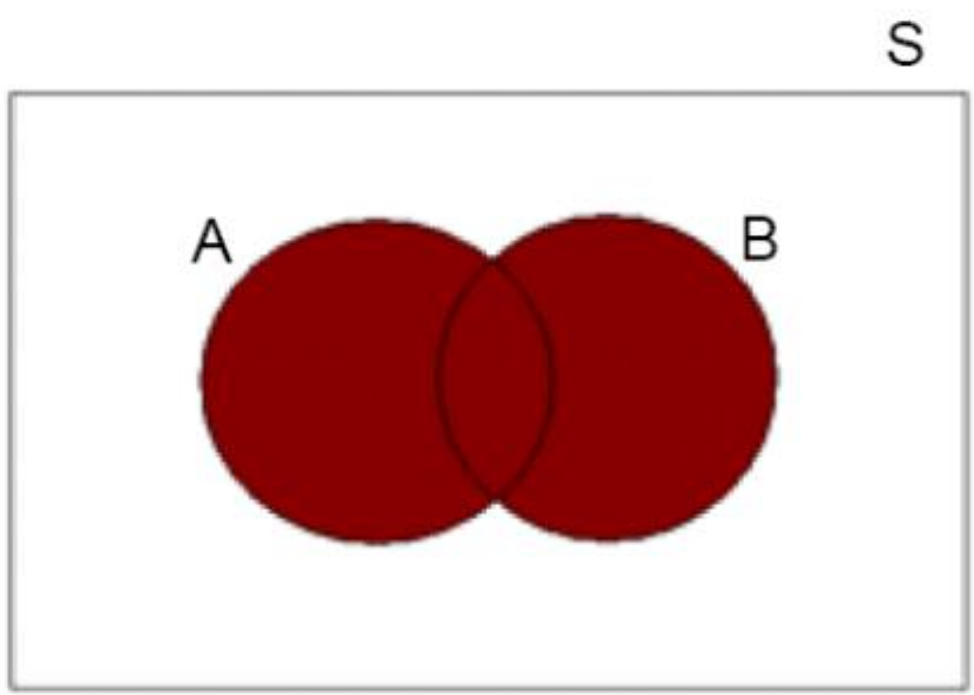
Given any two events (or sets) A and B, we have the following elementary set operations:

- The union
- The intersection
- The complement

Venn diagrams are often used to illustrate relationships between sets.

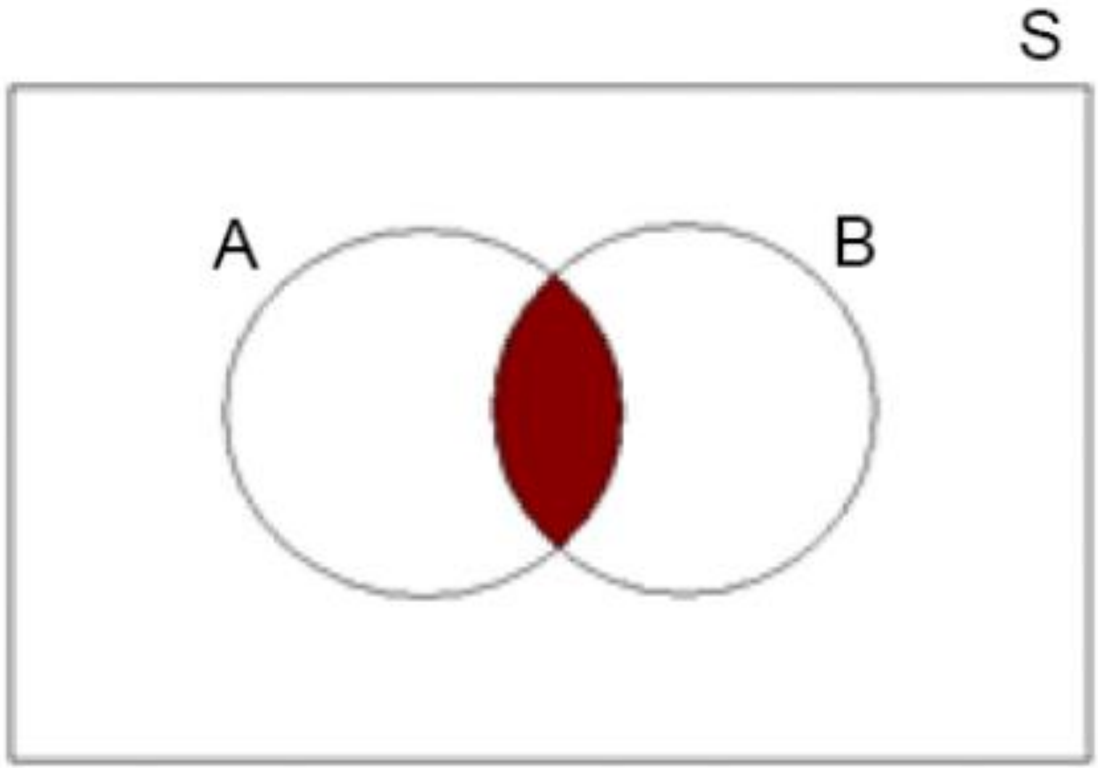
Union

The union of A and B, written as A U B and read "A or B", is the set of outcomes that belong to either A or B or both.



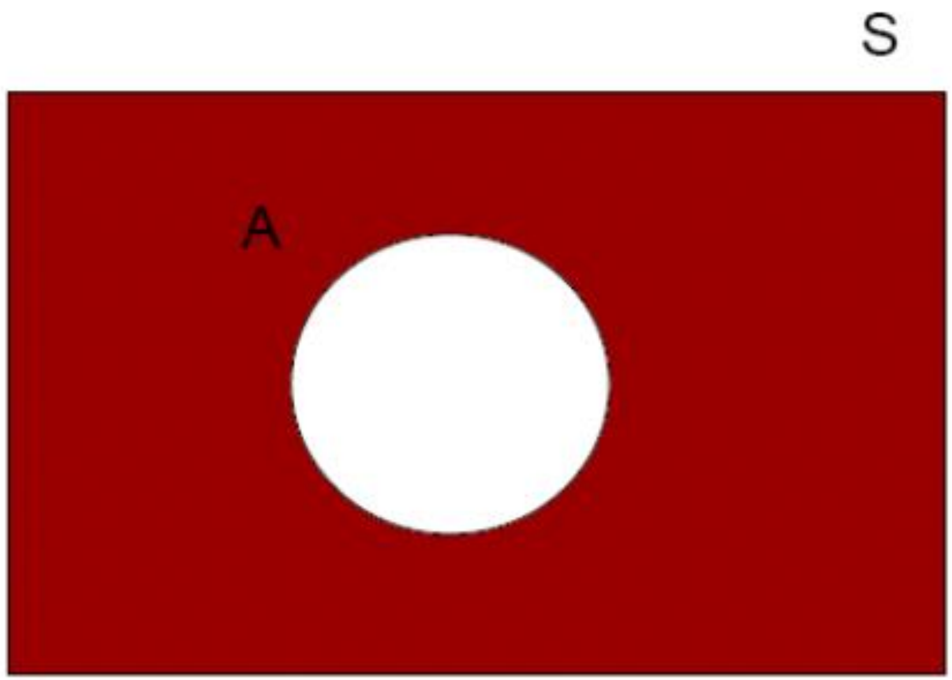
Intersection

- The intersection of A and B, written as A ∩ B, read "A and B", is the set of outcomes that belong to both A and B.



Complement

- The complement of A, written as A' or A^C, is the set of all outcomes in S that are not in A.



Example

Select a card at random from a standard deck of cards, and note its suit:

Clubs (Cl), Diamonds (D), Hearts (H) or Spades (Sp).

The sample space is S={Cl, D, H, Sp}.

Let: A={Cl, D}, B={D, H, Sp} and C={H}.

A U B = {Cl, D, H, Sp} = S

$A \cap B = \{D\}$

$A^C = \{H, Sp\}$

$A \cap C = \emptyset$ (null event – event consisting of no outcomes)

Disjoint events

- If $A \cap B = \emptyset$ then A and B are said to be mutually exclusive or disjoint events.

- Any event and its complement are disjoint!

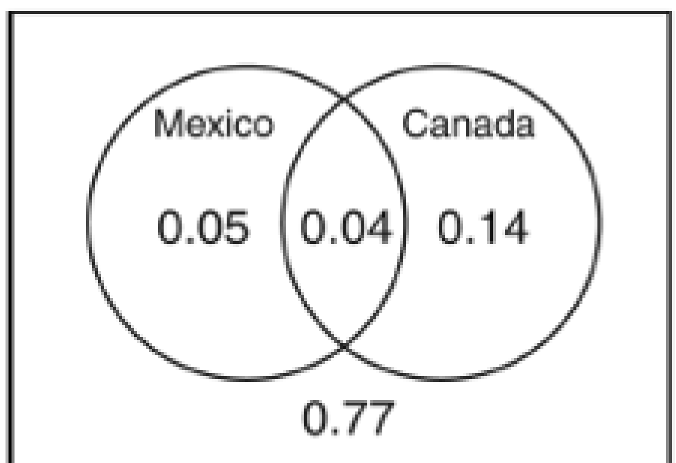
Question

Travel Suppose the probability that a U.S. resident has traveled to Canada is 0.18, to Mexico is 0.09, and to both countries is 0.04. What's the probability that an American chosen at random has

a) traveled to Canada but not Mexico?

b) traveled to either Canada or Mexico?

c) traveled to neither Canada or Mexico?



Probability models

- A probability model consists of a sample space (S) and the assignment of probabilities to each possible outcome.

- Probability that event A occurs is written as P(A), which will give a precise measure of the chance that A will occur.

Axioms of Probability

- To ensure the probability assignments will be consistent with our intuitive notions of probability, all assignments should satisfy the following axioms (basic properties) of probability.

1. For any event A, $P(A) \geq 0$.
2. $P(S) = 1$.
3. If $A_1, A_2, A_3, \ldots$ is an infinite (finite) collection of disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = \sum P(A_i)$$

Propositions

- For any event A, $0 \leq P(A) \leq 1$.

- $P(A) + P(A^C) = 1$.

- If event A is contained in event B, in the sense that every outcome in A is also in B, then $P(A) \leq P(B)$

- $P(\emptyset) = 0$

Consequences of the axioms

1. Complement rule: $P(A^c) = 1 - P(A)$

What is the probability of **not** rolling a pair of sixes in a roll of a pair of dice?

2. Difference rule: If $B \subseteq A$, then $P(A \setminus B) = P(A) - P(B)$ where *A* *B* refers to the *set difference between* A *and* B, *that* is, all the outcomes that are *A* but not in *B*.

3. Boole's (and Bonferroni's) inequality: generalization of the fact that the probability of the union of A and B is at most the sum of the probabilities.

Example (Exercise 1.4.5)

- Here's a question from Quora: "If a student applies to ten colleges with a 20% chance of being accepted to each, what are the chances that he will be accepted by at least one college?" Without making any further assumptions, what can you say about this chance?