Introduction – Basics

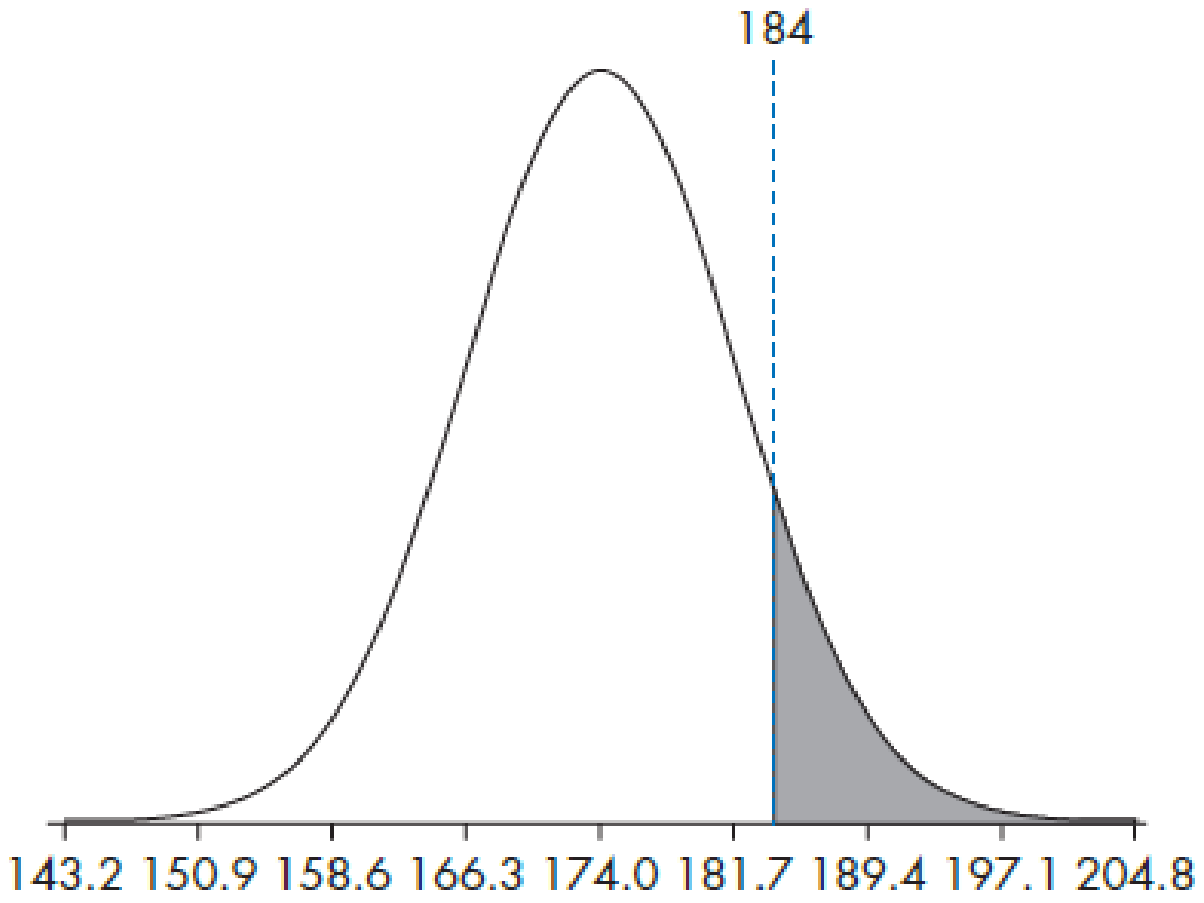
Section 1.1: Probabilities as Proportions

What is randomness?

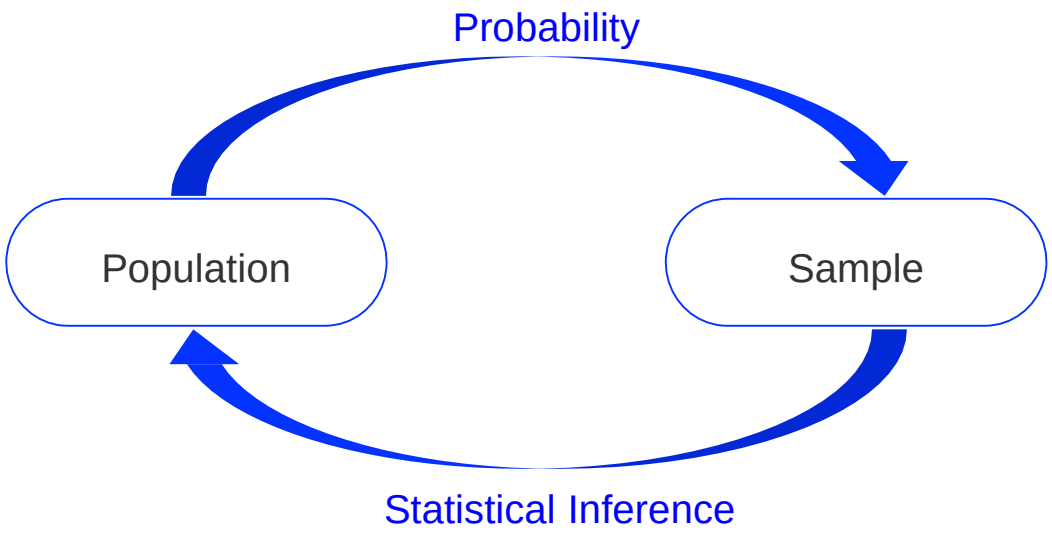
The world is full of random events that we seek to understand.

- An event is random if we know what outcomes could occur, but not the particular values that will happen.
- The outcome of these events is uncertain, but they follow a regular pattern.
- Probability theory is the mathematical representation of random phenomena.

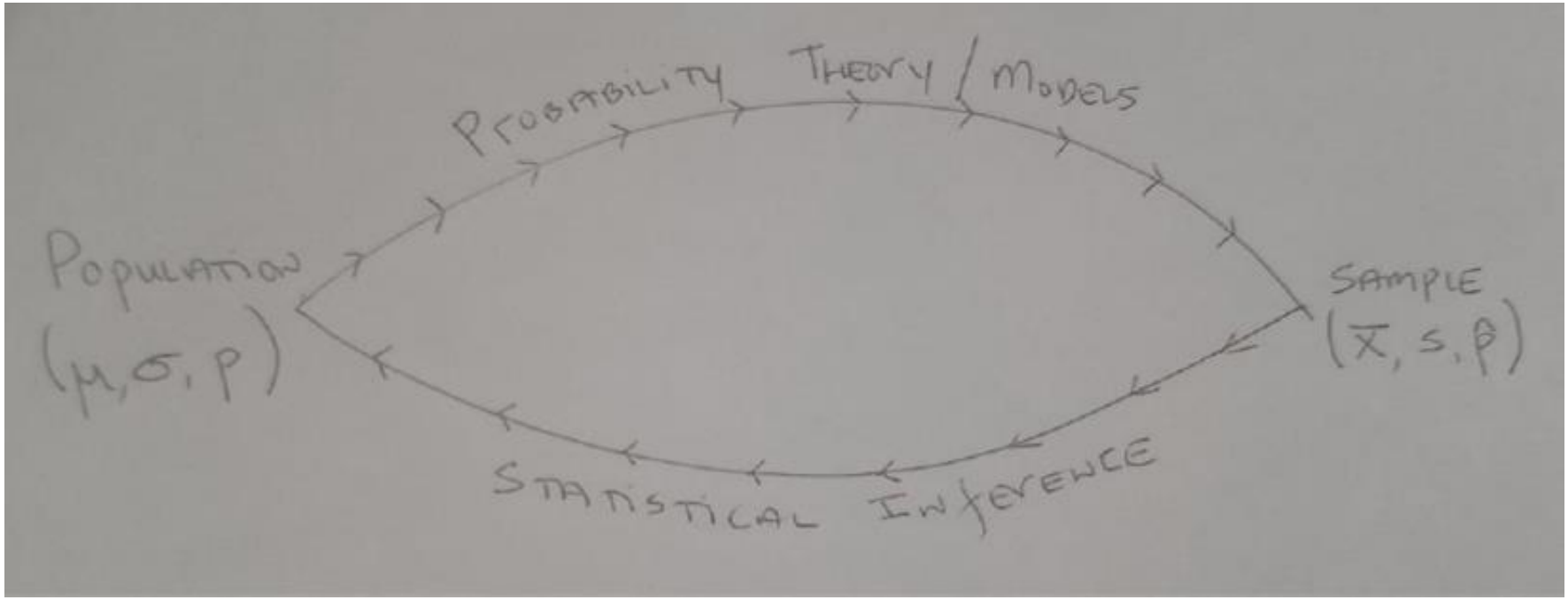
NHANES Men's Height



Probability and Statistical Inference



Probability and Statistical Inference



Terminology

An **experiment** is any action or process whose outcome is subject to uncertainty.

e.g. tossing a coin once or several times; selecting a card or cards from a deck; weighing a loaf of bread; etc.

The **sample space** of an experiment, denoted by **S**, is the set of all possible outcomes of that experiment.

Example. Flip a coin. Two possible outcomes: Heads (H) or Tails (T). S={H,T}.

Example: Toss a Die. What are the possible outcomes in the sample space?

Terminology

An event is any collection of possible outcomes, that is, any subset of S (including S itself).

- An event is **simple** if it consists of exactly one outcome and **compound** if it consists of more than one outcome.

- If the outcome of a random phenomenon is contained in an event A, then we say that A has occurred.

Examples

Example 1: Flip a coin twice. Four possible outcomes, S={HH, HT, TH, TT}.

- Let A be the event that we obtain at least one H in the two flips. A={HH, HT, TH}. **- compound event**

- Let B be the event that we obtain two H's in the two flips. B={HH}. – **simple event**

- **Example 2 -** Toss a die twice. S= {(1,1),(1,2),(1,3),…,(6,5),(6,6)}. Give an example of a simple event. A compound event

Computing probabilities

- If you have a well-shuffled deck of cards, and deal one card from the top, what is the chance of it being the queen of hearts? What is the chance that it is a queen (any suit)?
- How did you do this? What were your assumptions?
- Say we roll a die. What is S?
- What is the chance that the die shows a multiple of 3?

Computing Probabilities

Roll a fair die. S={1,2,3,4,5,6}. Our sample space consists of 6 points, each of which is **equally likely** to occur.

- P(roll a 1) =
- Let A = roll a 4 or less = {1,2,3,4}. P(A) =
- Let B = roll an even number = {2,4,6}. P(B) =

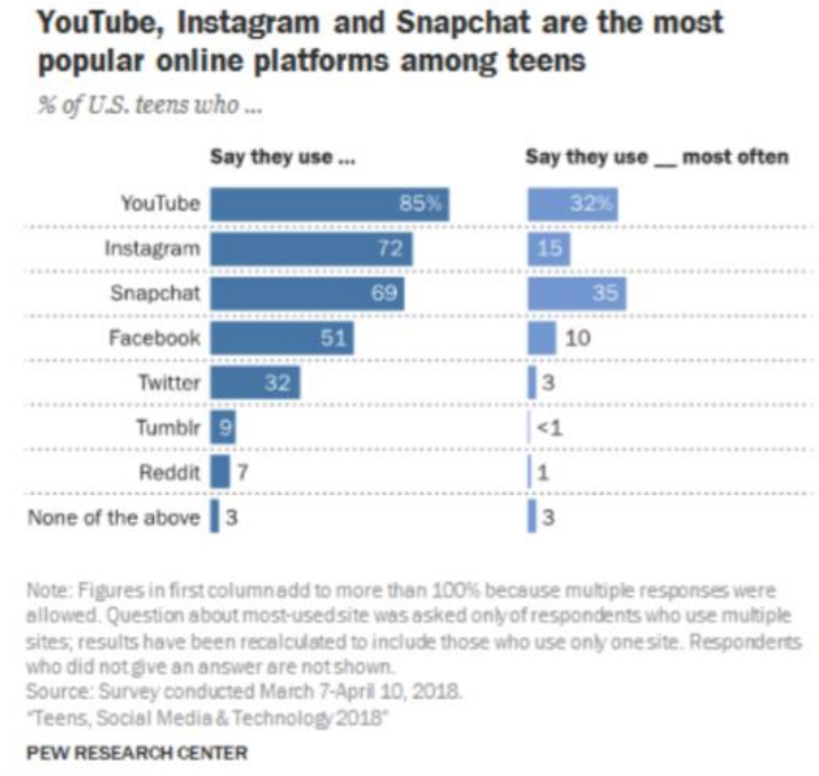
Chance of a Particular Outcome

We usually think of the chance of a particular outcome (roll a 6, coin lands heads etc) as the number of ways to get that outcome divided by the total possible number of outcomes.

$$\frac{\#\ of\ particular\ outcomes\ of\ interest}{total\ \#\ of\ outcomes\ possible}$$

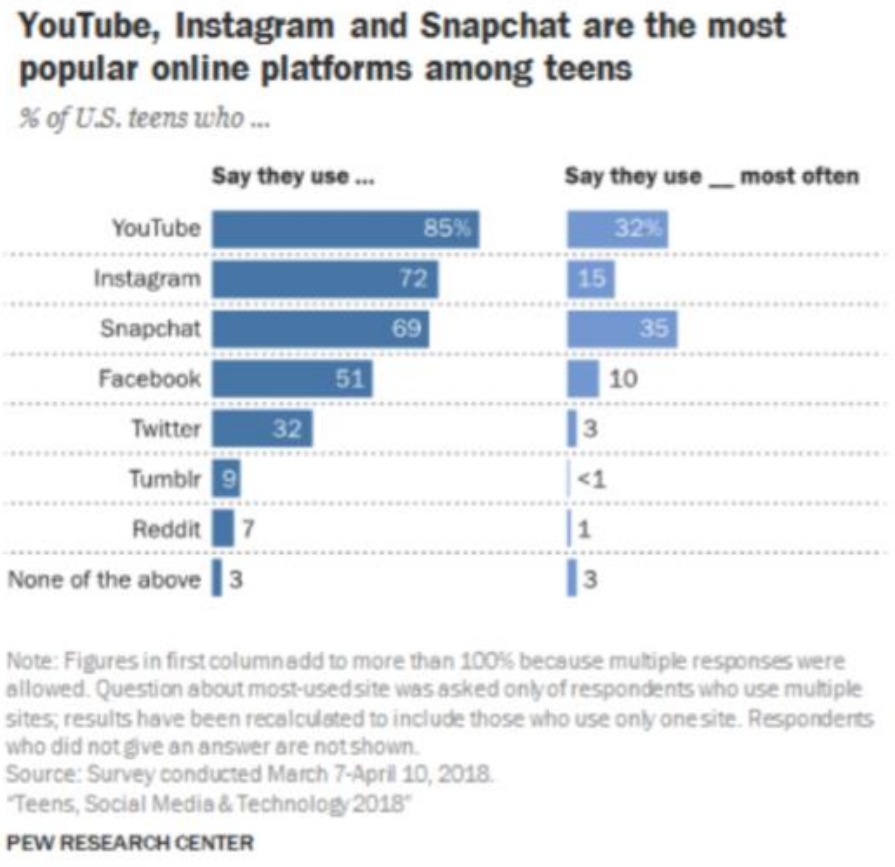
So if A is an event (subset of S), then $P(A) = \frac{\#(A)}{\#(S)}, A \subseteq S$

Not equally likely outcomes



- **Why do the % add up to more than 100 in the first graph?**

Not equally likely outcomes



1. What is the chance that a randomly picked teen uses FB most often?

2. What is the chance that a randomly picked teen did *not* use FB most often?

3. What is the chance that FB *or* Twitter was their favorite?

4. What is the chance that the teen used FB, just not most often?

5. Given that the teen used FB, what is the chance that they used it most often?

Conditional probability

- This probability we computed for #5 is called a ***conditional probability***. It puts a condition on the teen, and *changes* (restricts) the universe (the sample space) of the next outcome, a teen who likes FB best.

- To compute a conditional probability:
  - First restrict the set of all outcomes as well as the event to *only* the outcomes that *satisfy* the given condition
  - Then calculate proportions accordingly

Example

- A ten-sided fair die is rolled twice:

  - If the first roll lands on 1, what is the chance that the second roll lands on a number bigger than 1?

Q. How many possible outcomes in S?

Q. P(2nd roll > 1 | 1st roll = 1)?