STAT 88: Lecture 20

Contents

Warm up: Let the distribution of X be:

x	1	2	3
$(x-\mu_X)^2$			
$P(X=x)$	0.2	0.5	0.3

(a) Find $\mu_X = E(X)$.

(b) Find the distribution of $(X-\mu)^2$ in table.

(c) Find $E((X-\mu)^2)$.

6.1. Variance and Standard Deviation

Expectation : center of a distribution
Standard deviation : average spread of a dist'n about the center

Variance Let X be a random variable and let $\mu_X = E(X)$. Define $D = X - \mu_X$, the deviation from the expected value. Note $E(D) = E(X - \mu_X) = 0$.

We define a measure called the variance of X by

$$\mathrm{Var}(X) = E(D^2) = E((X - \mu)^2).$$

We saw how to calculate this in the warm up. Note that the units of X are squared.

Standard deviation

$$\mathrm{SD}(X) = \sqrt{\mathrm{Var}(X)} = \sqrt{E((X - \mu)^2)}.$$

Interpretation: "SD(X)" is roughly the "average" variation from the center.

Ex:

y	3	4	5
$P(Y = y)$	0.55	0.1	0.35

Calculate (1) $E(Y)$ (2) $\mathrm{Var}(Y)$ (3) $\mathrm{SD}(Y)$.

In Python:

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variance_table_Y
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y	(y - E(Y))**2	P(Y = y)
3	0.64	0.55
4	0.04	0.1
5	1.44	0.35

```
var_Y = sum(variance_table_Y.column(1) * variance_table_Y.column(2))
sd_Y = var_Y ** 0.5    ← (Var_Y)^½
sd_Y
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0.9273618495495703
```
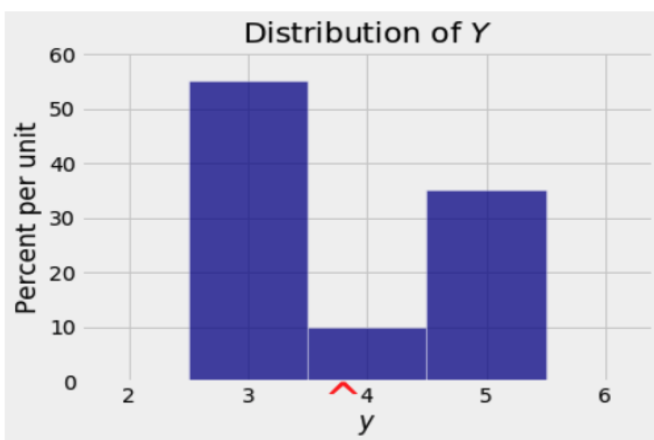
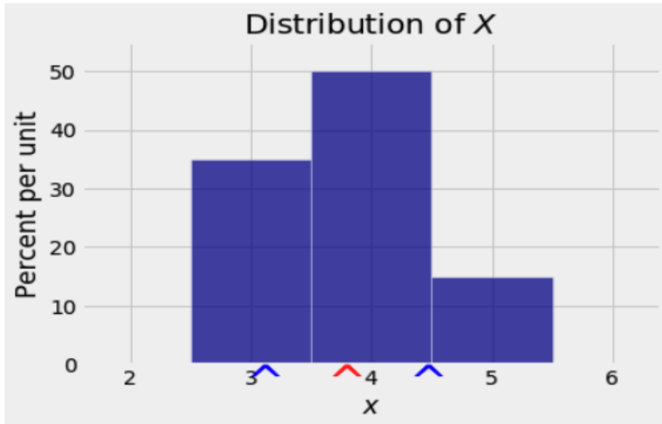

Picture:



E(Y) = 3.8

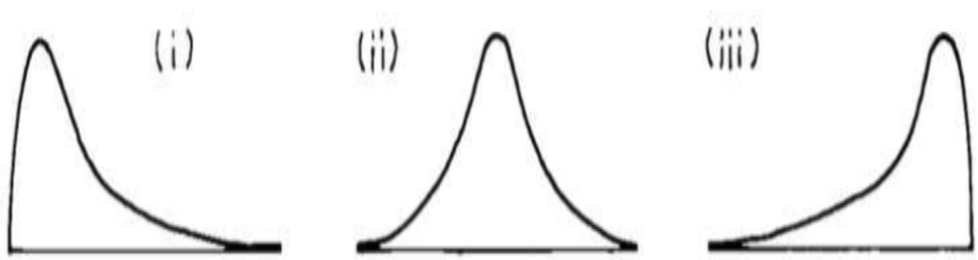
Compare with



$E(X) = 3.8$

SD(X) vs SD(Y) ?

Example: About 300 Stat 88 students at UC Berkeley, were asked how many college mathetmatics courses they had taken other than Stat 88. The average number of courses was about 1.1; the SD was about 1.5. Would the histogram for the data look like (i), (ii), or (iii)?



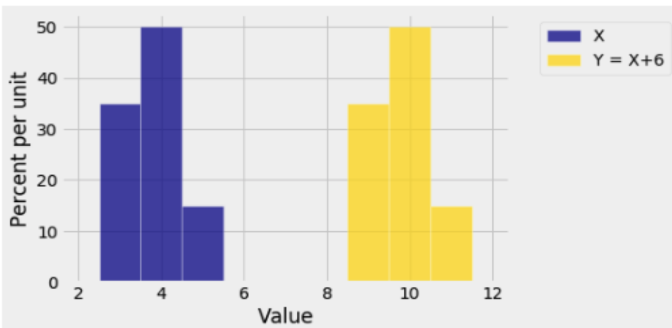
6.2. Simplifying the Calculation

Linear Transformations

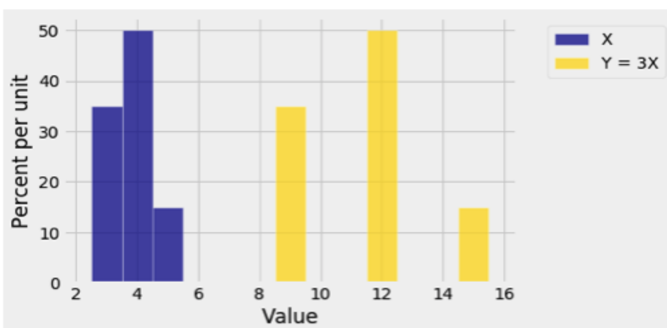
Celsius-Fahrenheit conversion:

$$Y = (9/5) \cdot X + 32.$$

How does SD(Y) compare to SD(X)?

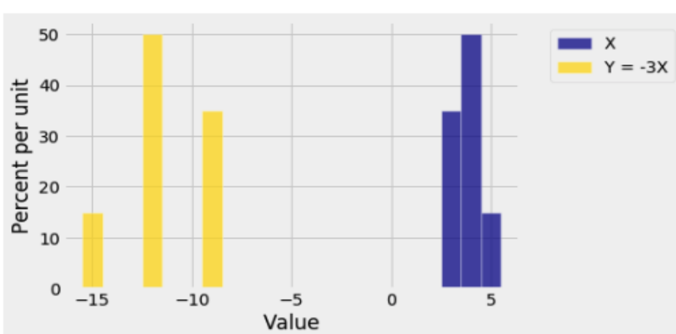


SD(x+b) = SD(X)



a>0

SD(aX) = a SD(X)



a<0,

SD(aX) = |a| SD(X)

So, we have

$$\mathrm{SD}(aX + b) = |a|\mathrm{SD}(X),$$

and

$$\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X).$$

Hence if $Y = (9/5)X + 32$, then

$$\mathrm{SD}(Y) = (9/5) \cdot \mathrm{SD}(X).$$

A Different Way of Calculating Variance An algebraic simplification for calculating variance:

$$\mathrm{Var}(X) = E((X - \mu_X)^2)$$

Ex:

y	3	4	5
$P(Y=y)$	0.55	0.1	0.35

Find $\text{Var}(Y) = E(Y^2) - E(Y)^2$.

Example: (Exercise 6.5.5) Let $p \in (0,1)$ and let X be the number of spots showing on a flattened die that shows its six faces according to the following chances:

- $P(X=1) = P(X=6)$
- $P(X=2) = P(X=3) = P(X=4) = P(X=5)$
- $P(X = 1 \text{ or } X = 6) = p$

Find $\text{SD}(X)$.