

# STAT 88: Lecture 5

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Section 2.4: Use and Interpretation

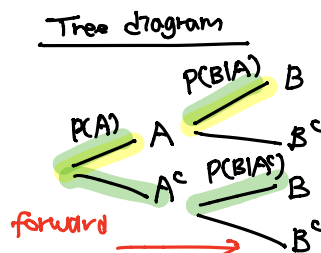
Section 2.5: Independence

## Last time

### Sec 2.3 Bayes' Rule:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}$$

- $P(A)$  is called **prior probability**;
- $P(B|A)$  is called **likelihood**;
- $P(A|B)$  is called **posterior probability**.



**Base rate fallacy** — according to Bayes' rule you can't ignore the base rate (i.e. prior probability) when computing the posterior probability.

**Warm up:** There are two boxes, the odd box containing 1 black marble and 3 white marbles, and the even bot containing 2 black marbles and 4 white marbles. A box is selected at random, and a marble is drawn at random from the selected box.

(a) What is the probability that the marble is black?

(b) Given that the marble is white, what is the probability that it came from the even box?

## 2.4. Use and Interpretation (continued)

Example: There are three boxes, each of which contains two coins. One box has two gold coins, one has two silver coins, and one has a gold coin and a silver coin. A box is picked at random and then a coin is picked at random from the box. Given that the coin is gold, what is the chance that the other coin in the box is silver?



Example: Around 2003, Sally Clarke, in a famous murder trial had two children one year apart who both died mysteriously. Sally Clarke's defence was that the babies both died of Sudden Infant Death Syndrome (SIDS),

- $A$  = event the first child dies of SIDS;
- $B$  = event the second child dies of SIDS.

Assume  $P(A) = P(B) = 1/8543$ . A medical expert witness said the chance of two babies dying of SIDS is  $1/8543^2 = 1/72982849 \approx 0$  and hence Sally Clarke must have murdered her babies.

What is the problem with argument?

There may be genetic or environmental factors that predispose families to SIDS so that the second case within the family becomes much more likely. Hence  $P(B|A) < 1/8543$  and  $P(A \cap B) = P(A)P(B|A) < 1/8543^2 = 1/72982849$ .

Example: A population consists of equal numbers of students in 4 Categories: freshman, sophomore, junior and senior. 4 people are drawn **with replacement** from the population. What is the chance that a member from each category is chosen?

Example: (Exercise 2.6.5) There are  $n$  students in a class. Assume that each student's birthday is equally likely to be any of 365 days of the year, regardless of the birthdays of others.

- (a) What is the chance that at least one of the students was born on January 1?
- (b) What is the chance that at least two students have the same birthday?

Example: Suppose  $A$  and  $B$  are two events with

$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

For what values of  $P(B)$  would  $A$  and  $B$  be independent?

1.  $P(B) = 0$
2.  $P(B) = 0.3$
3.  $P(B) = 0.6$
4. none of the above