

STAT 88: Lecture 3

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Sec 1.3 Fundamental Rules

Three axioms of probability:

1. $P(A) \geq 0$ for all $A \subseteq \Omega$.
2. $P(\Omega) = 1$.
3. **Addition Rule.** If A and B are mutually exclusive, i.e. $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$.

Based on the axiom, we have **upper bound** and **lower bound** as (why?)

$$\left(\max\{P(A), P(B)\} \leq P(A \cup B) \leq P(A) + P(B). \right) \quad (1)$$

Textbook notation : **AB means $A \cap B$.** In the lecture note, we will use **$A \cap B$** to denote the intersection.

De Morgan's laws: $(A^c \cup B^c)^c = A \cap B$. Then

$$P(A \cap B) = P((A^c \cup B^c)^c) = 1 - P(A^c \cup B^c).$$

We know from (1) that $\max\{P(A^c), P(B^c)\} \leq P(A^c \cup B^c) \leq P(A^c) + P(B^c)$, therefore

$$\left(1 - P(A^c) - P(B^c) \leq P(A \cap B) \leq 1 - \max\{P(A^c), P(B^c)\}. \right)$$

Warm up: (Exercise 1.4.5) If a student applies to ten colleges with a 20% chance of being accepted to each, what are the chances that he will be accepted by at least one college? Be clear about any assumptions you are making.

2.1. The Chance of an Intersection

Pick two cards at random without replacement from a deck that contains one red, one blue, and one green card (R, B, G). Find

$$P(\text{1st Card B and 2nd Card R}).$$

The outcome space $\Omega = \{RB, RG, BG, BR, GR, GB\}$. The draws are at random, so all six pairs are **equally likely**. Then

$$P(\text{1st Card B and 2nd Card R}) = P(\{BR\}) = \frac{1}{6}.$$

Alternatively, we can use the multiplication rule:

$$P(\{BR\}) = \underbrace{P(\text{2nd Card R}|\text{1st Card B})}_{=1/2} \cdot \underbrace{P(\text{1st Card B})}_{=1/3}.$$

Multiplication rule For $A, B \subseteq \Omega$,

$$P(A \cap B) = P(A|B)P(B).$$

Since $P(A \cap B) = P(B \cap A)$, we also have $P(A \cap B) = P(B \cap A) = P(B|A)P(A)$.

Example: What is chance 1st card in a 52 card deck is queen **“and”** the last is queen?

Inclusion-Exclusion For $A, B \subseteq \Omega$,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Example: What is chance 1st card in a 52 card deck is queen **“or”** the last is queen?

Example: A deck of cards is shuffled. What is the chance that the top card is the king of spades or the bottom card is the king of spades?

Intersection of Several Events Consider a poker hand (5 cards randomly drawn without replacement).

What is the chance all 5 cards are hearts?

What is the chance that all five cards are of the same suit?

2.2. Symmetry in Simple Random Sampling

Sampling individuals at random without replacement is one of the most natural ways to collect a random sample from a finite population. It is called simple random sampling and will be studied extensively in this course. We will examine simple random sampling in the context of dealing hands of cards from a deck (or population) of size 52.

Recall a deck of 52 cards has 4 suits.



How many possible pairs of cards are there?

If you deal 2 cards, what is the chance the 2nd card is red?